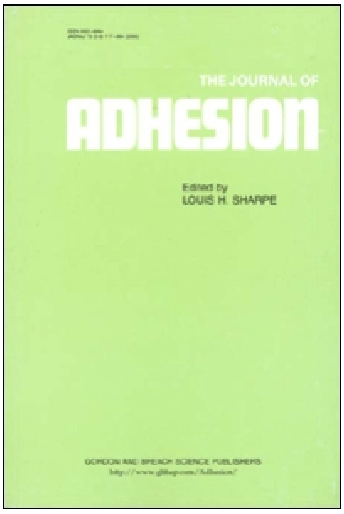

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Re-evaluation of Adhesive Fracture Energy†

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A model to evaluate the Energy Release Rate (ERR) of adhesives using the Double Cantilever Beam (DCB) specimen is described. The model accounts for the adhesive bond thickness and its material properties. The analysis, considered as an improvement to the built-in cantilever beam model, treats the adherend as a finite beam which is partly free and partly supported by an elastic foundation and the adhesive bond as a thin strip under prescribed displacement. The results show significant effect of the adhesive parameters on the total ERR and that the built-in cantilever model underestimates the ERR. In general, the contribution of the adhesive bond to the ERR increases for softer adhesives, shorter cracks and thicker bonds.

KEY WORDS Fracture energy; bond thickness; modulus; adhesives; double cantilever beam specimen; modeling.

INTRODUCTION

The critical energy release rate, referred to as fracture energy, G_{1c}, is commonly used to characterize the strength of adhesive joints.

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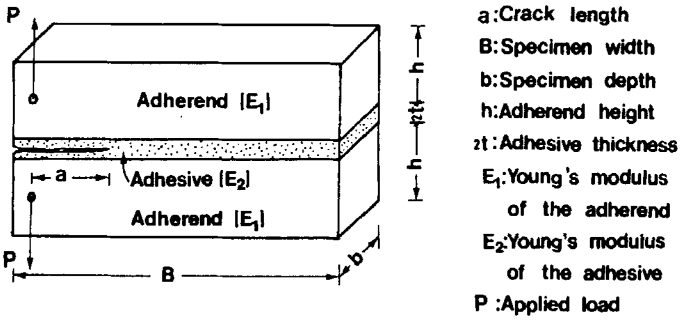

FIGURE 1 Specimen geometry of the SDCB.

This energy is measured using a Sandwich-type Double Cantilever Beam (SDCB) specimen, in which a layer of the adhesive is inserted between prismatic adherends (Figure 1).

The idea of a monolithic Double Cantilever Beam (DCB) was originated in 1930 by Obreimoff[1] and a simple analysis was developed later by Benbow and Roesler[2] and by Gilman.[3] Each arm, separated from its mate by a crack, is treated as a cantilever beam built-in at one end and only bending of the cantilever part was considered in the energy analysis. Later, Kanninen[4] modelled each half of the DCB as a beam partly free and partly supported by an elastic foundation. This foundation represents the interaction of the two beams along the crack trajectory in the uncleaved portion. Gates[5] further generalized Kanninen's model by accounting for shear stresses and noticed that this effect is significant in the case of a short DCB when the uncleaved ligament is comparable to the beam height.

The DCB specimen was adapted to evaluate the energy of delamination in composite materials by several researchers[6–8] and the fracture energy in structural adhesives by Ripling and Mostovoy.[9–11] The built-in cantilever beam model is used to analyze the SDCB specimen (Figure 2a). Only bending and shear deflection of the cantilever part was considered and the expression for the fracture energy is presented by Ripling and Mostovoy[9–11] as:

$$G_{1c} = \frac{4P_c^2}{E_1 b^2}\left[\frac{3a_c^2}{h^3} + \frac{1}{h}\right] \tag{1}$$

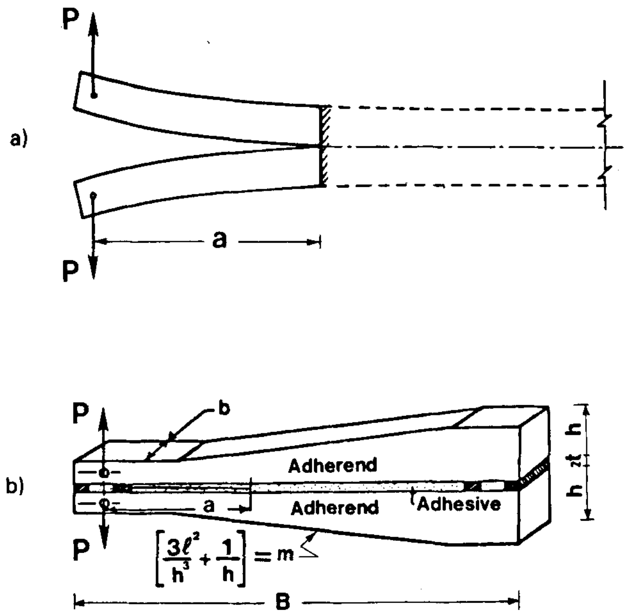

FIGURE 2 a) Built-in cantilever beam model.
b) Adhesive bond.

where P_c and a_c are the applied load and the crack length at which the specimen failed. E_1 is the elastic modulus of the adherend, h is its height and b is the depth of the specimen (Figure 1). Measurements of both critical load and critical crack length are required in order to utilize this expression. If the SDCB is designed such that the term $[3a^2/h^3 + 1/h]$ is a constant, however, only the critical load measurement is needed. Such a design is known as Tapered (or Contoured) Double Cantilever Beam (TDCB)[10–11] (Figure 2b).

Equation 1 does not account explicitly for the properties of the adhesive (elastic modulus and Poisson's ratio) or for its thickness despite the experimentally observed dependency of the fracture energy on these parameters.[12–15] Furthermore, it was found that the deflection at the point of load application is larger than predicted by this model.[9–11] Hence, a fitting parameter was introduced empirically as an extra crack length a_0 $(=0.6\,h)$ to account for the deflection due to rotation at the assumed built-in end of the beam (crack front). Nevertheless, the measurement of fracture energy based on Eq. (1) or on the TDCB design does not explicitly describe the adhesive bond performance.

This paper introduces a new model for the Energy Release Rate (ERR) evaluation. The model accounts for the thickness of the bond and its elastic properties. Detailed stress analysis upon which the model is founded has been reported recently.[16,17]

DESCRIPTION OF THE MODEL

Consider the SDCB shown in Figure 1. Under normal applied load P (mode I), a crack in the adhesive bond advances along the plane of symmetry. Since the adhesive bond is usually softer and thinner than the adherend, the system of each arm of the adherend and the adhesive bond can be modelled as a Beam on Elastic Foundation (BEF) (Figure 3). The initial crack renders the beam partially free and partially supported by an elastic foundation (Figure 3a). This technique, similar to Kanninen's model, is also considered by Chow *et al.*[18] The stretched bond is modelled as a thin strip under prescribed displacement (Figure 3b). This displacement is determined by the deflection of the adherend beam.

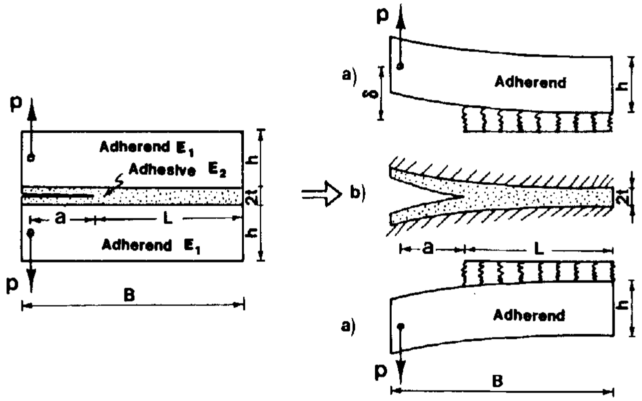

FIGURE 3 Schematic decomposition of the SDCB
a) Adherend beam
b) Adhesive bond.

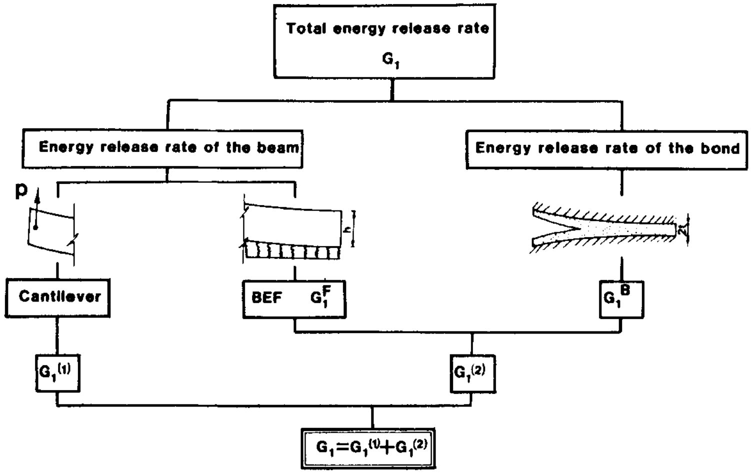

FIGURE 4 Flow chart diagram of the decomposition of the ERR.

The total ERR of the adherend–adhesive system G_1 is the sum of the ERR due to the stretched bond G_1^B and the ERR of the two adherend beams. The latter is again decomposed into the ERR $G_1^{(1)}$ of the cantilever part and the ERR of the uncleaved portion of the beam supported by the elastic foundation G_1^F. This scheme is summarized in the flow chart in Figure 4. The ERR associated with the adhesive bond $G_1^{(2)}$ is then presented as:

$$G_1^{(2)} = G_1^F + G_1^B \tag{2}$$

The total ERR is, therefore, the sum of the ERR of the cantilever part $G_1^{(1)}$ and the ERR associated with the adhesive bond $G_1^{(2)}$, i.e.,

$$G_1 = G_1^{(1)} + G_1^{(2)} \tag{3}$$

ANALYSIS AND DISCUSSION

The free part of the beam is treated as a built-in cantilever beam of length equal to the crack length. Ignoring the shear effect, the ERR of the two cantilever beams corresponds to the Ripling-Mostovoy model and was found to be:[16,17]

$$G_1^{(1)} = \frac{12P^2a^2}{E_1b^2h^3} \tag{4}$$

Following our previous work[16,17] the ERR associated with the adhesive bond is given by:

$$G_1^{(2)} = G_1^{(1)}[\Phi^2 + \alpha\psi^2 - 1] \tag{5}$$

where

$$\Phi = \left[\frac{\text{Sinh}^2 \beta L + \sin^2 \beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L}\right] + \frac{1}{\beta a}\left[\frac{\text{Sinh}\,\beta L\,\text{Cosh}\,\beta L - \sin\beta L\cos\beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L}\right]$$

$$\psi = \sin\beta L\,\text{Sinh}\,\beta L - \frac{1}{\beta a}\left(\frac{\zeta - 1}{2}\right)\sin\beta L\,\text{Cosh}\,\beta L$$
$$+ \Phi\cos\beta L\,\text{Cosh}\,\beta L - \frac{1}{\beta a}\left(\frac{\zeta + 1}{2}\right)\cos\beta L\,\text{Sinh}\,\beta L$$

Here

$$\zeta = \left[\frac{\text{Sinh}^2 \beta L + \sin^2 \beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L}\right] + 2\beta a\left[\frac{\text{Sinh}\,\beta L\,\text{Cosh}\,\beta L + \sin\beta L\cos\beta L}{\text{Sinh}^2 \beta L - \sin^2 \beta L}\right]$$

$\alpha = (1 - \nu_2)/(1 + \nu_2)(1 - 2\nu_2)$, ν_2 is the Poisson's ratio of the adhesive, β is a parameter of dimension $(\text{length})^{-1}$ and is equal to $\left[\frac{3E_2/E_1}{th^3}\right]^{1/4}$. L is the width of the uncracked specimen, i.e., $L = B - a$, where B is the total specimen width (Figure 1).

The total ERR of the specimen which accounts for the ERR of the two beams and that associated with the adhesive bond is found by adding Eqs. (4) and (5) to obtain:

$$G_1 = G_1^{(1)}[\Phi^2 + \alpha\psi^2] \tag{6}$$

where Φ, α and ψ are given above. This expression accounts for the adhesive bond performance and should be used for fracture energy evaluation.

The ratio $G_1/G_1^{(1)}$ of the actual (total) ERR of the adhesive DCB specimen (Eq. (6)) to that obtained from Eq. (4) is considered in terms of the bond thickness and the elastic constants of the adhesive and the adherend. These parameters are expressed by the dimen-

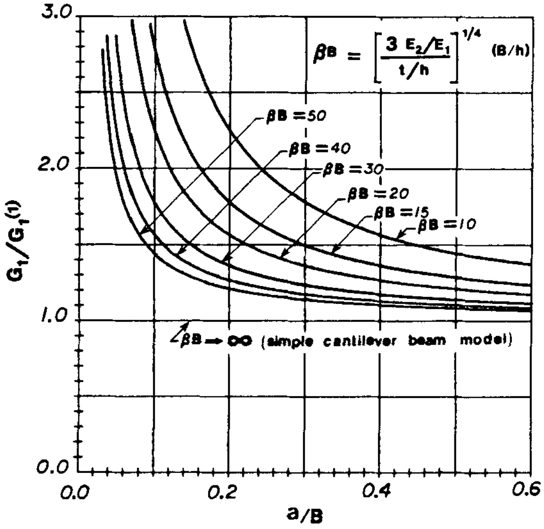

FIGURE 5 Ratio of the total ERR G_1 and the ERR of the cantilever parts $G_1^{(1)}$ *versus* the crack length a normalized by the specimen width B for various constant βB.

sionless term:

$$\beta B = \left[\frac{3(E_2/E_1)}{(t/h)}\right]^{1/4}(B/h) \tag{7}$$

A plot of $G_1/G_1^{(1)}$ for various values of βB *versus* crack length is shown in Figure 5. Obviously, the fracture energy (ERR) could be significantly underestimated if computed conventionally,[18] particularly for short crack length and low βB. The discrepancy between G_1 and $G_1^{(1)}$ becomes insignificant for very high βB and/or very large crack length. Such conditions, however, seriously diminish the contribution of the adhesive bond to the overall joint performance which is primarily sought. The magnitude of the βB term is generally controlled by the ratio B/h. Thus, if the tested adhesive is to be employed for very thin structures, i.e., B/h is very large (>50), then G_1 approaches $G_1^{(1)}$ and may be computed from the conventional beam theory.[20] In many structural applications, however, thick adherends are employed and the test geometry must be designed accordingly. On the other hand, for a very small B/h ratio and/or a very short crack the ratio $G_1/G_1^{(1)}$ tends to infinity. In view of these two limitations and in order to assess the bond contribution to the overall joint performance appropriately, specimen dimensions should be selected to fall within an appropriate range, i.e., $10 < \beta B < 50$ (Figure 5).

Effect of bond thickness

To illustrate the effect of bond thickness on the total ERR we consider a specimen with a width-to-height ratio (B/h) of 10. The ratio of the elastic modulus of the adherend E_1 to that of the adhesive E_2 is about 30, which approximates that of Al/Epoxy systems. The adhesive Poisson's ratio ν_2 is taken as 0.35. Plots of $G_1/G_1^{(1)}$ as a function of the bond thickness t normalized with respect to the adherend height h is shown in Figure 6 for various normalized crack lengths (a/B). For the conventional DCB model G_1 equals $G_1^{(1)}$ and is independent of the bond thickness (dotted line). The total ERR G_1 according to our model could reach more

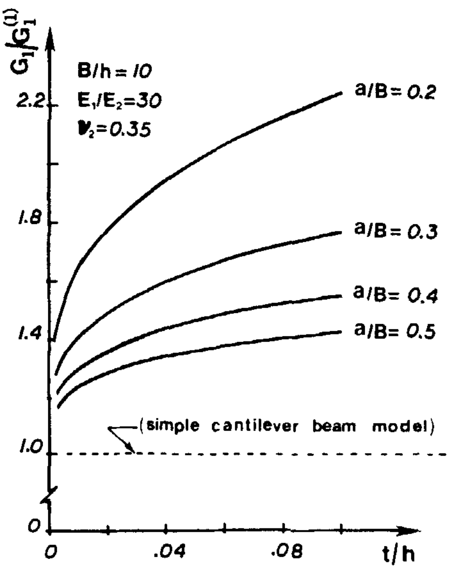

FIGURE 6 Contribution of the bond thickness to the total ERR for various crack lengths.

than twice that estimated from the conventional model depending on the crack length and the bond thickness. The shorter the crack, the stronger is the dependency of the total ERR on the bond thickness.

Effect of the adhesive modulus

An adhesive bond made from softer material would allow for larger deformation, which yields more ERR. This effect is illustrated in Figure 7 which shows the dependency of $G_1/G_1^{(1)}$ on the relative stiffness for the case of $t/h = 0.1$ and for various crack lengths. The contribution of the adhesive bond is larger for shorter cracks and, of course, for softer adhesives. An increase in Poisson's ratio from 0.35 for a rigid adhesive to 0.45 for a softer adhesive corresponds to an additional increase of the ERR of about 5%. In addition, it is interesting to note that for a monolithic DCB (i.e., $E_1/E_2 = 1$) G_1 remains higher than $G_1^{(1)}$. This effect, according to Kanninen's analysis, is rationalized by the notion that one beam acts as an elastic foundation for the other.

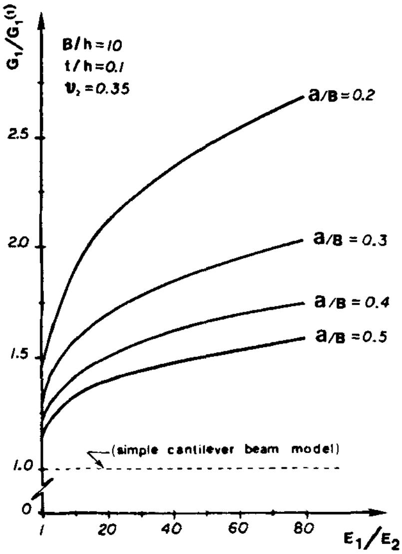

FIGURE 7 Effect of the relative moduli E_1/E_2 on the total ERR for various crack lengths.

CALCULATION PROCEDURE

In this section we outline a simplified procedure to compute the ERR for the DCB adhesive specimen according to the proposed model. The ASTM flat DCB test specimen[19] is considered for illustration. The chosen dimensions are presented in Figure 8. This type of geometry is easier to prepare than that of the TDCB specimens. The flat DCB requires the measurement of the critical crack length. The fracture energy may be calculated according to either of the following procedures:

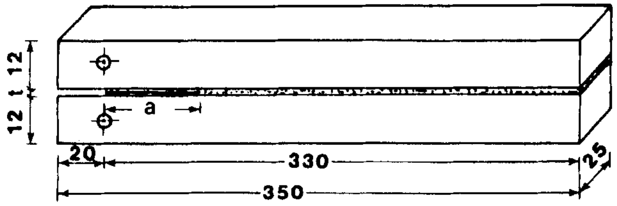

FIGURE 8 Flat ASTM standard specimen. All dimensions are in mm.

1) Use Eq. (6) together with Eqs. (5) and (4).

2) Evaluate the term βB and knowing the ratio a_c/B get the ratio $G_{1c}/G_{1c}^{(1)}$ from Figure 5 by interpolation if necessary. The value found should be multiplied by the G_{1c} of Eq. (4).

3) It is also possible to use the simple computer program outlined in the appendix. An example is given with the listing file. Such a program can be performed by a Personal Computer. The evaluation of the fracture energy can be obtained after the input of the necessary data.

CONCLUSION

The adhesive fracture energy G_{1c}, as evaluated using the Ripling-Mostovoy built-in cantilever model, is underestimated because the contribution of the adhesive bond is ignored. The present model is refined by considering the adherend as a beam which is partly free and partly supported by an elastic foundation, and the adhesive bond as a thin strip under prescribed displacement. The model accounts for the thickness of the adhesive bond and its elastic properties. It is shown that the contribution of the adhesive bond is larger for shorter cracks, softer adhesives and thicker bonds and may reach more than twice the value obtained by the built-in model.

CLOSING REMARK

The energy release rate formalism presented in this paper is obviously founded on elastic considerations. It is, however, recognized that most structural adhesives develop a "plastic" zone at the tip of a crack.[13] Thus, the application of the elastic foundation idea may be limited and require further refinement to account for such "plasticity," the effect of which ought to be examined experimentally. Nevertheless, the present formalism illustrates the effects of the adhesive modulus and thickness, and improves the accuracy of fracture energy calculations particularly at short crack lengths.

Acknowledgment

The authors wish to acknowledge the financial support of the Center for Adhesives, Sealants and Coatings (CASC) at Case Western Reserve University.

Appendix

```
c  *******************************************
c****  This routine is designed to compute the    *
c****  energy Release Rate (ERR) for the Double   *
c****  Cantilever Beam adhesive joint. It uses    *
c****  the equations described in this paper.     *
c****                                             *
c****    NOTATION                                 *
c****      E1=Elastic Modulus of Adherend         *
c****      E2=Elastic Modulus of Adhesive         *
c****    PR2=Poisson's Ratio of the Adhesive      *
```

```
c****      B=Specimen Width                          *
c****     bd=Specimen Depth                          *
c****      h=Adherend Height                         *
c****      t=Adhesive Thickness                      *
c****      P=Applied Load                            *
c****     CL=Crack Length                            *
c****    G11=ERR of the cantilever                   *
c****     G1=Total ERR                               *
c ******************************************************

      IMPLICIT REAL*8(A-H, O-Z)
C
C      Input the Material Properties
C
       WRITE(*,*)' Give Material Properties'
       WRITE(*,*)' E1,E2,PR2'
       READ(*,*)E1,E2,PR2
       WRITE(*,*)' Give Specimen Geometry'
       WRITE(*,*)' B,bd,h,t'
       READ(*,*)B,bd,h,t
       WRITE(*,*)' Give Experimental Data'
       WRITE(*,*)' P,CL'
       READ(*,*)P,CL
C
C      Compute Necessary Ratios
C
       BH=B/h
       E=E1/E2
       TH=t/(2.0*h)
       RLB=CL/B
C
C      Compute necessary constants
C
       BB=((3.0/(TH*E))**0.25)*BH
       BL=RLB*BB
       BC=BB-BL
       AL=(1.0-PR2)/((1.0+PR2)*(1.0-2.0*PR2))
C
C      Compute the necessary functions
C
```

```
      S=DSIN(BC)
      C=DCOS(BC)
      SH=DSINH(BC)
      CH=DCOSH(BC)
C
C
      D0=SH*SH-S*S
      D1=(SH*SH+S*S)/D0
      D2=(SH*CH+S*C)/D0
      D3=(SH*CH-S*C)/D0
      D4=D1+2.0*BL*D2
      PHI=(BL*D1+D3)
      PSI=BL*S*SH-((D4-1.0)/2.0)*S*CH
      PSI=PSI+(PHI*C*CH)-((D4+1.0)/2.0)*C*SH
      PHI=PHI/BL
      PSI=PSI/BL
      F=PHI**2+AL*(PSI**2)
C
C     Compute the ERR of the cantilever
C
      G11=(12.0*P*P*CL*CL)/(E1*bd*bd*h*h*h)
C
C     Compute the TOTAL ERR and difference
C
      G1=G11*F
C
C     Display the output
C
      WRITE(*,0010)E1,E2,PR2,B,bd,h,t
 0100  FORMAT(
     1 '         Material Properties',//
     2 '         Elastic Moduli',/
     3 '  Adherend=',g15.3,'  Adhesive=',g15.3,/
     4 '    Adhesive Poisson's Ratio=',g10.5,///
     5 '         Specimen Geometry',//
     6 '    Width=',g12.5,'    Depth=',g12.5,/
     7 '    Adherend height=',g12.5,
     8 '  Adhesive thickness=',g12.5)
      WRITE(*,0020)P,CL,G11,G1,F
 0020  FORMAT(///
```

```
    1 '         Experimental Data',//
    2 '  Load=',g12.5,'  Crack Length=',g12.6,//
    3 '      ****ENERGY RELEASE RATES****',/
    4 '    Cantilever=',g12.5,'  Total ERR=',
    5 g12.5,/,'   Difference of ',g12.5, 'times')
      END
```

Example: ASTM flat specimen
Input data

```
  Give Material Properties
  E1,E2,PR2
.7E+11,.3E+10,0.35
  Give Specimen Geometry
  B,bd,h,t
0.33,0.025,0.012,0.0006
  Give Experimental Data
  P,CL
100.,0.05
```

Output

```
                    Material Properties
                       Elastic Moduli
  Adherend=    .700E+11  Adhesive=    .300E+10
         Adhesive Poissons Ratio=.35000

                     Specimen Geometry
      Width= .33000      Depth= .25000E-01
            Adherend height= .12000E-01
          Adhesive thickness= .60000E-03

                     Experimental data
    Load=100.00      Crack Length=.500000E-01

             **** ENERGY RELEASE RATES ****
    Cantilever=3.9683        Total ERR=5.3339
            Difference of  1.3441  times
```

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